Starter Questions

Solve

$$\log_5(n^2 + 20n) = 3$$

Solve these simultaneous equations.

$$\log x + \log y = 2$$

$$2^{x-6} = 4^{y+2}$$

Q Kinematics

Q4

Use calculus in kinematics for motion in a straight line: $v=\frac{\mathrm{d}r}{\mathrm{d}t}$, $a=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}^2r}{\mathrm{d}t^2}$, $r=\int v\,\mathrm{d}t$, $v=\int a\,\mathrm{d}t$; extend to 2 dimensions using vectors.

Assessed at AS and A-level

Teaching guidance

Students should:

- know and be able to apply the following to motion in a straight line:

$$v=\frac{\mathrm{d}r}{\mathrm{d}t} \qquad a=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}^2r}{\mathrm{d}t^2}$$

$$r=\int v\,\mathrm{d}t \qquad v=\int a\,\mathrm{d}t$$

- explore the relationship between calculus methods and the constant acceleration equations.

7.4 Motion for variable acceleration

If the acceleration is not constant (variable), then the constant acceleration equations **cannot** be used.

Velocity = gradient of s-t graph
(rate of change of displacement over time)

Acceleration = gradient of v-t graph
(rate of change of velocity over time)

7.4 Motion for variable acceleration

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Example 1a

An object has displacement, (in km) at time, (in hours) given by the equation: , where

a) Calculate the object's initial velocity:

Initial velocity

7.4 Motion for variable acceleration

Example 1b

An object has displacement, (in km) at time, (in hours) given by the equation: , where

b) Find the object's acceleration when t = 3 hours

When :

7.4 Motion for variable acceleration

Example 2a

The acceleration of an object, in kmh^{-2}, at time hours is given by

Given that its initial velocity is 1 kmh^{-1}, find:

a) Its velocity after 3 hours

When :

7.4 Motion for variable acceleration

Example 2a

The acceleration of an object, in kmh^{-2}, at time hours is given by

Given that its initial velocity is 1 kmh^{-1}, find:

a) Its velocity after 3 hours

When :

7.4 Motion for variable acceleration

Example 2b

The acceleration of an object, in kmh^{-2}, at time hours is given by

Given that its initial velocity is 1 kmh^{-1}, find:
b) Its displacement after 2 hours

When :

7.4 Motion for variable acceleration

Example 2b

The acceleration of an object, in kmh^{-2}, at time hours is given by

Given that its initial velocity is 1 kmh^{-1}, find:
b) Its displacement after 2 hours

When :

7.4 Motion for variable acceleration

Example 3

At time t(s), an object's displacement s(cm) follows the function:
Find the time at which the object reaches its maximum velocity.

7.4 Motion for variable acceleration

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At time t(s), an object's displacement s(cm) follows the function:
Find the time at which the object reaches its maximum velocity.
Velocity is a maximum when

7.4 Motion for variable acceleration

Example 3

At time t(s), an object's displacement s(cm) follows the function:
Find the time at which the object reaches its maximum velocity.
Check maximum:

When ,
When ,
Hence, the object reaches maximum velocity when

7.4 Motion for variable acceleration

Example 4

A particle moves with acceleration ms^{-2}. It is initially at a point O and is travelling with velocity 2 ms^{-1}. Show that its direction of travel changes twice and find the distance between the points where this occurs.

NB: a particle changes direction when ms^{-1}

7.4 Motion for variable acceleration

Example 4

A particle moves with acceleration ms^{-2}. It is initially at a point O and is travelling with velocity 2 ms^{-1}. Show that its direction of travel changes twice and find the distance between the points where this occurs.

When :

7.4 Motion for variable acceleration

Example 4

A particle moves with acceleration ms^{-2}. It is initially at a point O and is travelling with velocity 2 ms^{-1}. Show that its direction of travel changes twice and find the distance between the points where this occurs.

Times when particle changes direction:

7.4 Motion for variable acceleration

Example 4

A particle moves with acceleration ms^{-2}. It is initially at a point O and is travelling with velocity 2 ms^{-1}. Show that its direction of travel changes twice and find the distance between the points where this occurs.

So, from 0 to when , the particle is moving forwards.
From to , the particle is moving backwards.
Then from the particle moves forwards again.
Hence, the particle changes its direction of travel at and , twice.

7.4 Motion for variable acceleration

Example 4

A particle moves with acceleration ms^{-2}. It is initially at a point O and is travelling with velocity 2 ms^{-1}. Show that its direction of travel changes twice and **find the distance between the points where this occurs**.

When :

7.4 Motion for variable acceleration

Example 4

A particle moves with acceleration ms^{-2}. It is initially at a point O and is travelling with velocity 2 ms^{-1}. Show that its direction of travel changes twice and **find the distance between the points where this occurs**.

When :

When :

$\therefore$ distance between the points where the particle changes direction is

7.4 Motion for variable acceleration

Example 5

A particle moves in a straight line with constant acceleration, $a\,\text{m s}^{-2}$. Given that its initial velocity is $u\,\text{m s}^{-1}$ and its initial displacement is 0 m, prove that:

a its velocity, $v\,\text{m s}^{-1}$ at time t s is given by $v = u + at$

b its displacement, s m, at time t is given by $s = ut + \frac{1}{2}at^2$

Sub in ICs:

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